Viscous Flow on Deforming Overset Grids

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Thanks to: Bill Henshaw, Mike Shelley

Overset 2002

Outline

- ✓ Overview of methods for interfacial flows
- ✓ A flapping filament in 2D incompressible flow
  - Modeling the filament
  - Body-fitted grid generation
  - Overview of “OverBlown”, our N-S solver
  - Some numerical results

Overview of methods for interfacial flows

- Methods are typically *front-tracking*…
  - Boundary integral methods
  - The immersed boundary method
  - FronTier (Glimm et al)
- … or *front-capturing*
  - Shock capturing schemes (not the focus here)
  - Arbitrary-Lagrangian-Eulerian (common at LLNL)
  - Volume-of-fluid methods
  - Level set methods

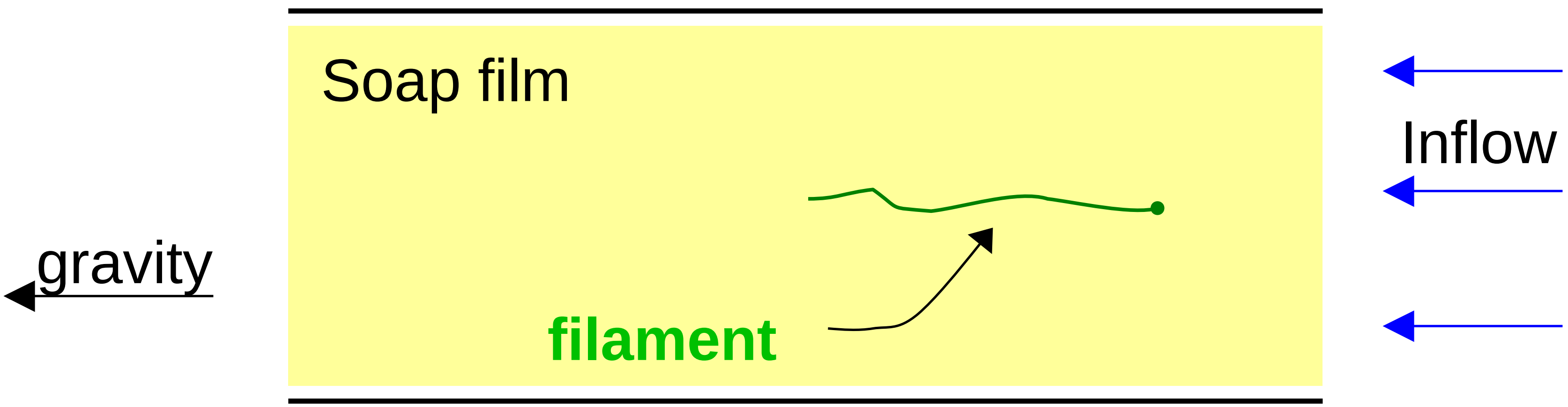
Front-tracking methods

- Boundary integral methods
  - E. g. Hou, Lowengrub & Shelley (JCP ’94) & …
  - Need Stokes flow, or irrot.&incompressible flow
  - Spectral accuracy possible
  - Reduced dimensionality… used by many…
- Immersed boundary method (McQueen & Peskin)
  - Cartesian grid for N-S solver → FAST!
  - Interfacial forces enter as body force in N-S
  - …used by many
- FronTier (Glimm *et al*); Tryggvason & Unverdi.
  - Finite-element mesh for the surface
  - Allows changes in topology (with a lot of work…)

Front capturing methods

- ✓ Volume-of-fluid methods
  - 2 fluids, track volume fraction ϕ, ***interface is a jump in*** $\boldsymbol{\phi}$
  - Simple Line Interface Calculation(Noh&Woodward'76)
  - VOF (Nichols&Hirt '75, '81)
  - 2^{nd} order version (Pilliod & Puckett, Li & Zaleski)
  - ***Large density jump is a problem for elliptic solve***
- ✓ Level set methods (Osher, Sethian)
  - Track a smooth "level set function" ϕ, ***interface is*** $\boldsymbol{\phi}$***=0***
  - ***Problems with mass conservation***
- ✓ Hybrid schemes
  - Zhilin Li, Osher *et al* for Hele-Shaw flow
  - Sussman & Puckett, Level set/volume of fluid method

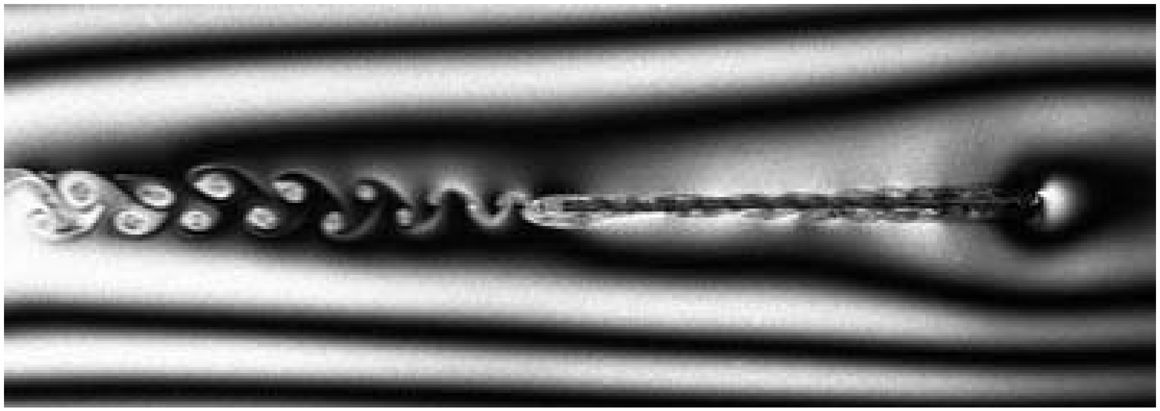
Dynamics of an elastic filament in soap film flow



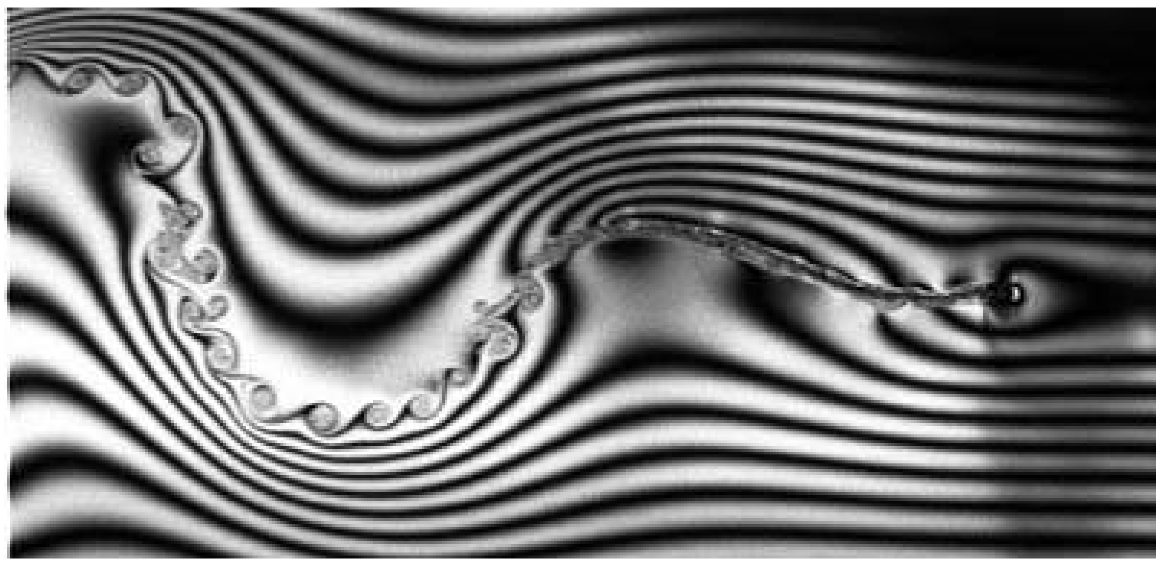
WITH:
Bill Henshaw
Mike Shelley
LLNL&NYU

- Experimental realization of 2D flow (Couder et el, Phys. D, 1989)
- Flow is governed by 2D incompressible NS with thickness='density'
- Elastic filament couples to the flow & has dynamics
- Prototype of fluid-structure interactions
- Motivated by insect flight
  - Elastic, moving & deformable wings in viscous flow
- PROBLEMS: moving, complex geom.; stiffness from elasticity

Experiments: Filament in a soap film flow



Flat state



Flapping state

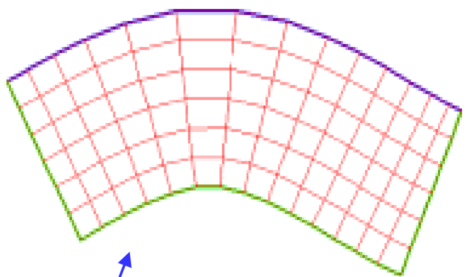
→ At higher flow rate

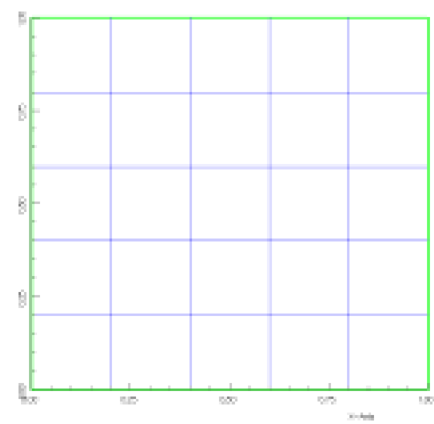
("1D flag in a 2D flow", Zhang, Childress, Libchaber, Shelley, Nature, Dec. 2000)

Transforming PDEs to complex geometry

“Physical” coords

“Transformed” coords





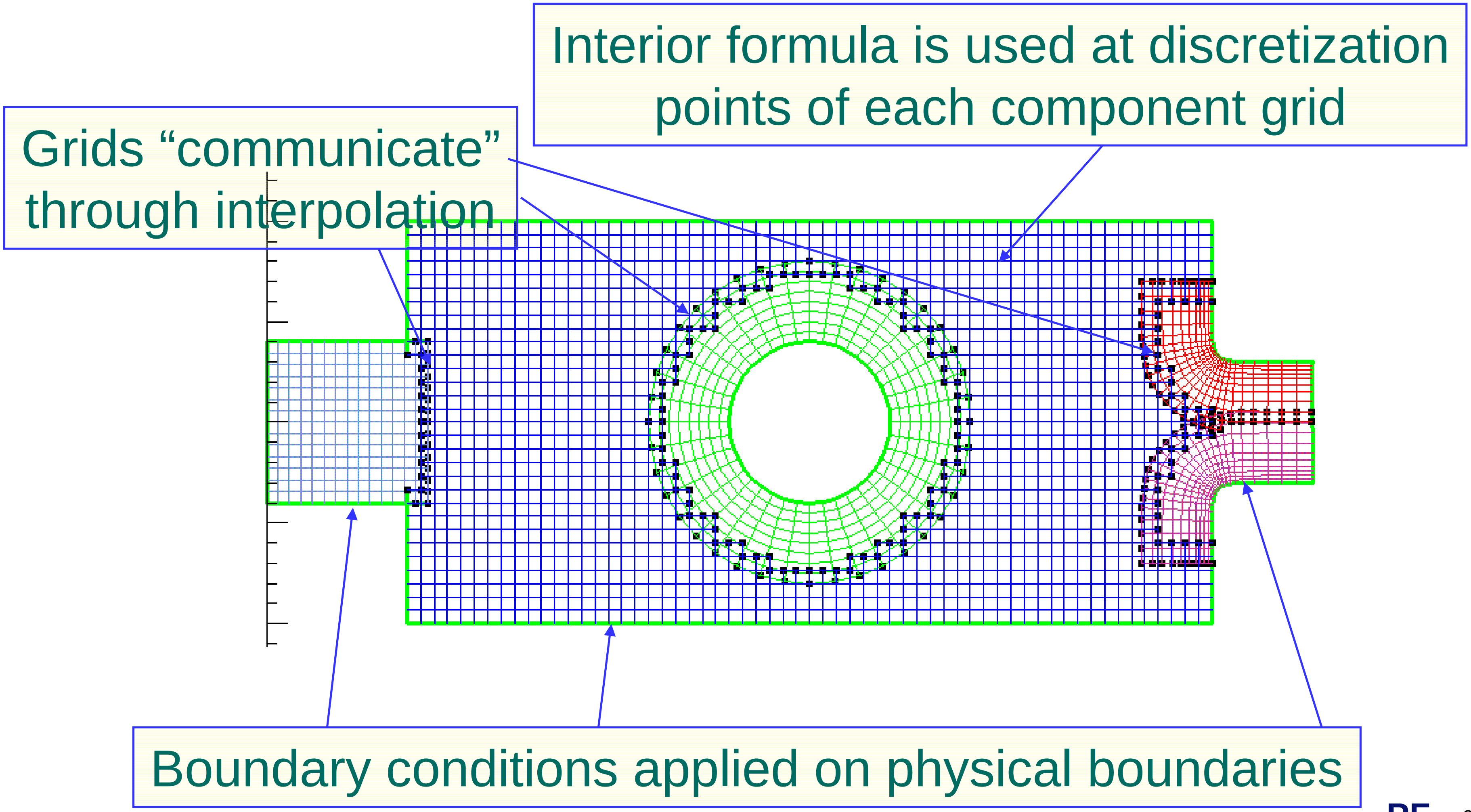
$$\mathbf{x} = \mathbf{g}(\mathbf{r})$$

Transformation mapping

$$\mathbf{x} = (x, y)$$

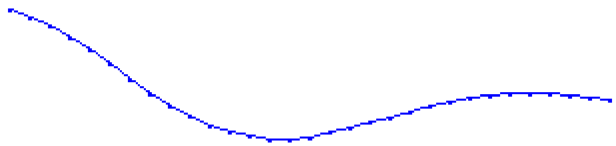
$$\mathbf{r} = (r_1, r_2) = (r, s)$$

Solving PDEs on overlapping grids

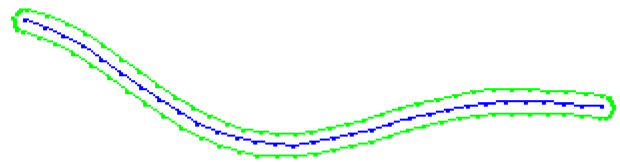


Modeling the filament: Body-fitted grid

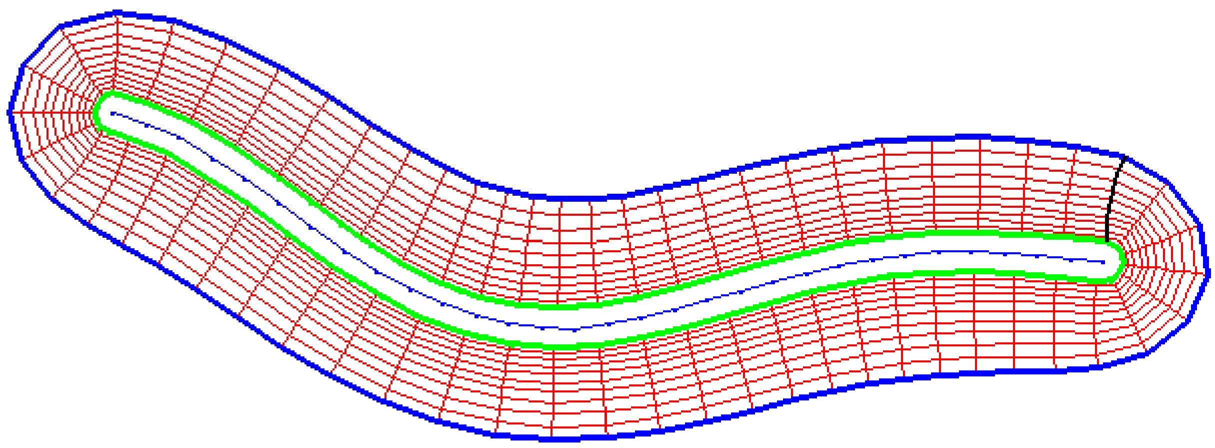
(Elastic) filament



Thick filament



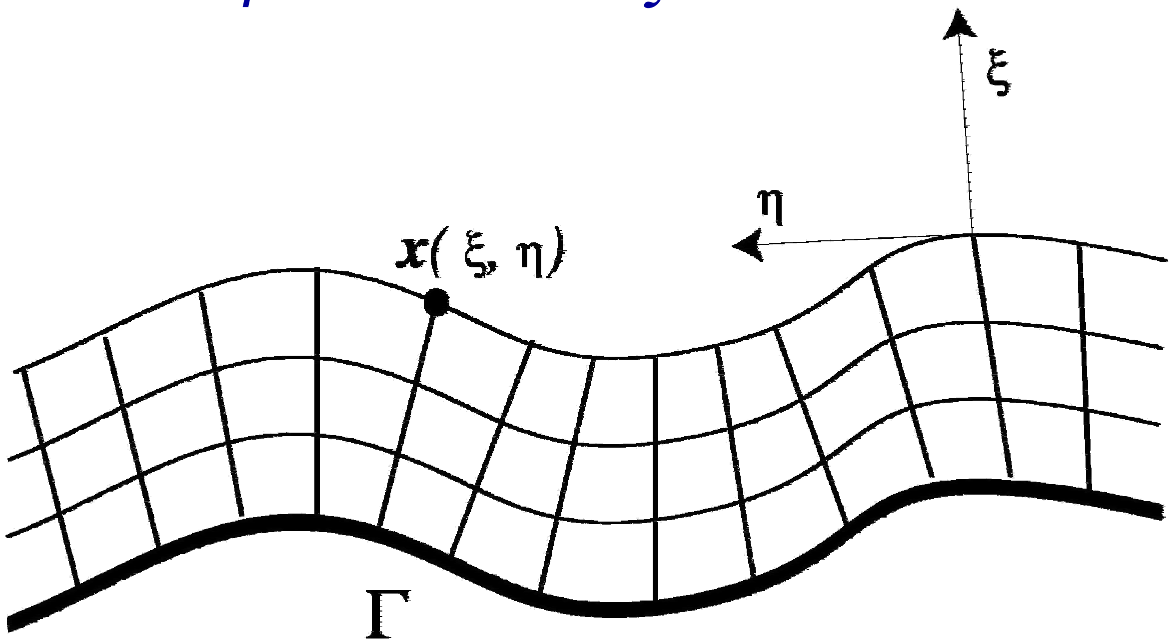
Filament + body-fitted grid



(Only pre-determined motion of the filament is considered)

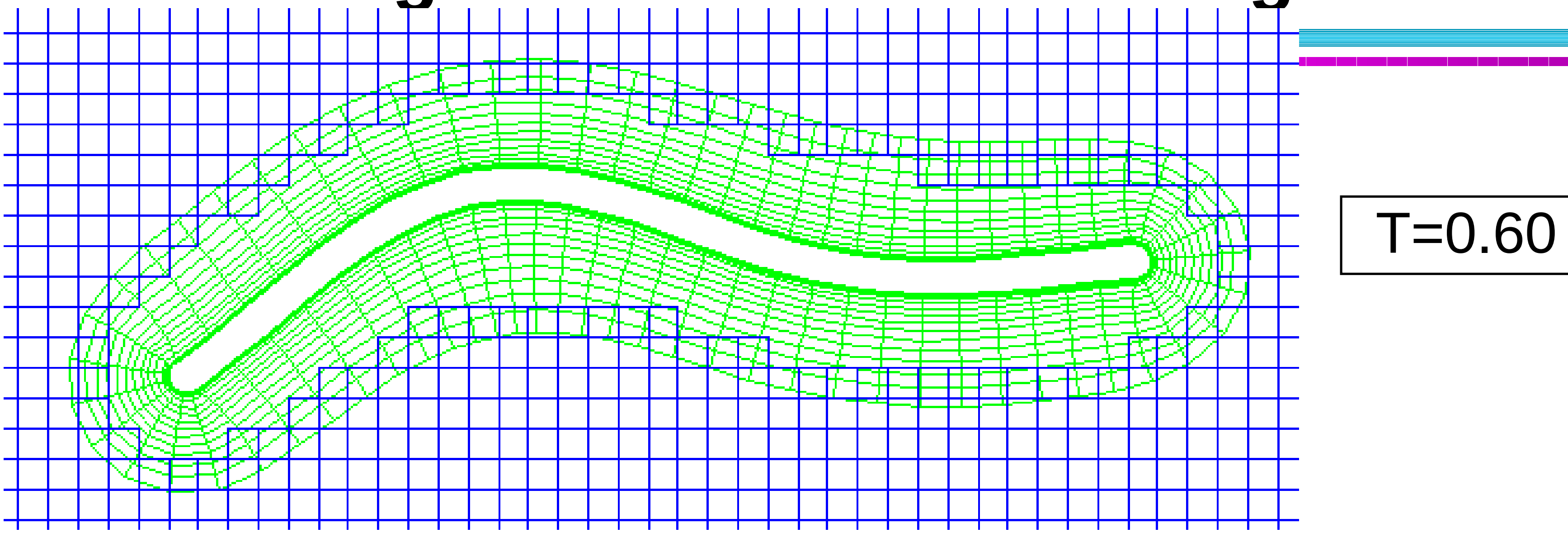
Grid generation – body fitted grid

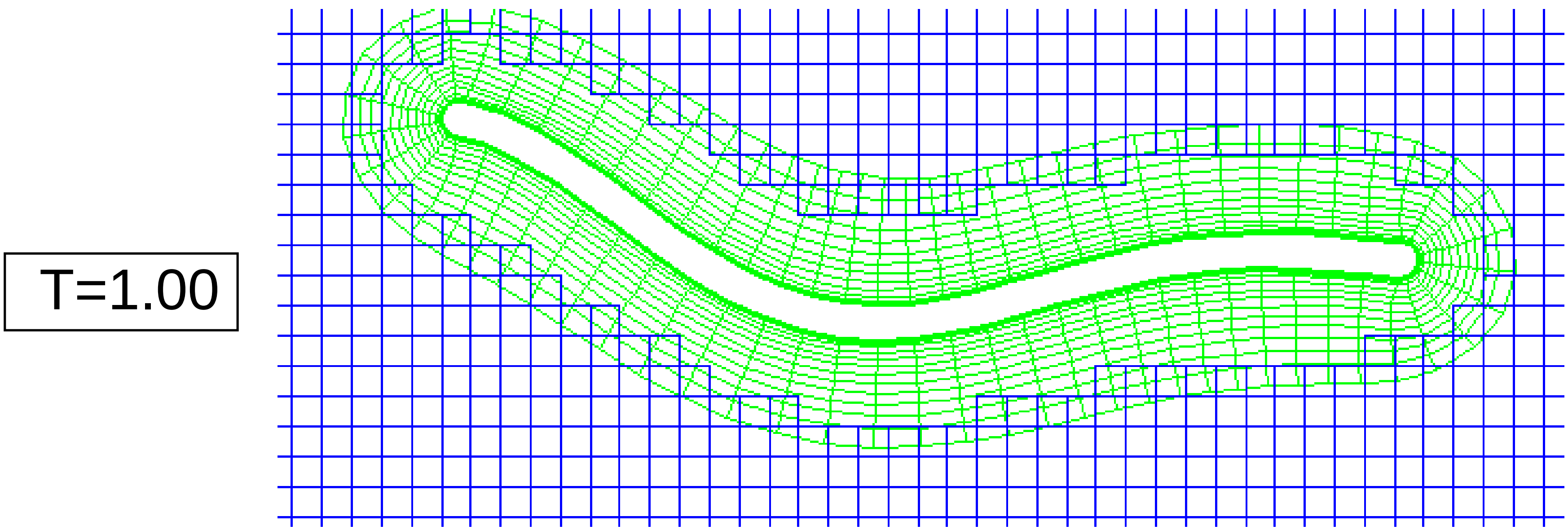
March in "pseudo-time" ξ from the interface Γ



- Hyperbolic grid generation of moving grids, each timestep
- Uses Overture Hyperbolic grid generator (Henshaw)
- Methods similar to *W. Chan et al.*

Modeling the filament: Overset grid





Discretization of the incompressible Navier-Stokes equations

- ✓ Pressure-velocity formulation (Henshaw '94)

$$\begin{aligned} \mathbf{u}_t + (\mathbf{u}\cdot\nabla)\mathbf{u} + \nabla p - \nu\Delta\mathbf{u} &= 0 \\ \Delta p - (\nabla u \cdot \mathbf{u}_x + \nabla v \cdot \mathbf{u}_y + \nabla w \cdot \mathbf{u}_z) - C_d(\nu)\nabla\cdot\mathbf{u} &= 0. \end{aligned}$$

- ✓ Time-stepping, method-of-lines
  - Adams-Bashfort-Moulton PC, 2nd order (explicit)
  - Crank-Nicholson on viscous term(implicit, some grids)
- ✓ Spatial discretization of the momentum equation
  - Non-conservative 2nd order accurate finite differences

Pressure-velocity formulation on moving grids

A moving component grid is defined by a time-dependent mapping

$$\mathbf{x} = \mathbf{G}(\mathbf{r}, t)$$

The convection term has a grid velocity contribution

$$\begin{aligned} \mathbf{U}_t + [(\mathbf{U} - \underline{\dot{\mathbf{G}}}) \cdot \tilde{\nabla}]\mathbf{U} + \tilde{\nabla} P &= \nu \tilde{\Delta} \mathbf{U}, \\ \tilde{\Delta} P + \sum_i \tilde{\nabla} U_i \cdot \partial_{x_i} \mathbf{U} &= 0, \end{aligned}$$

(Henshaw '94, F & Henshaw '01, part of OverBlown)

Boundary conditions: moving grid

Let the boundary be defined by $\mathbf{x} = \mathbf{G}(\mathbf{r}, t)$

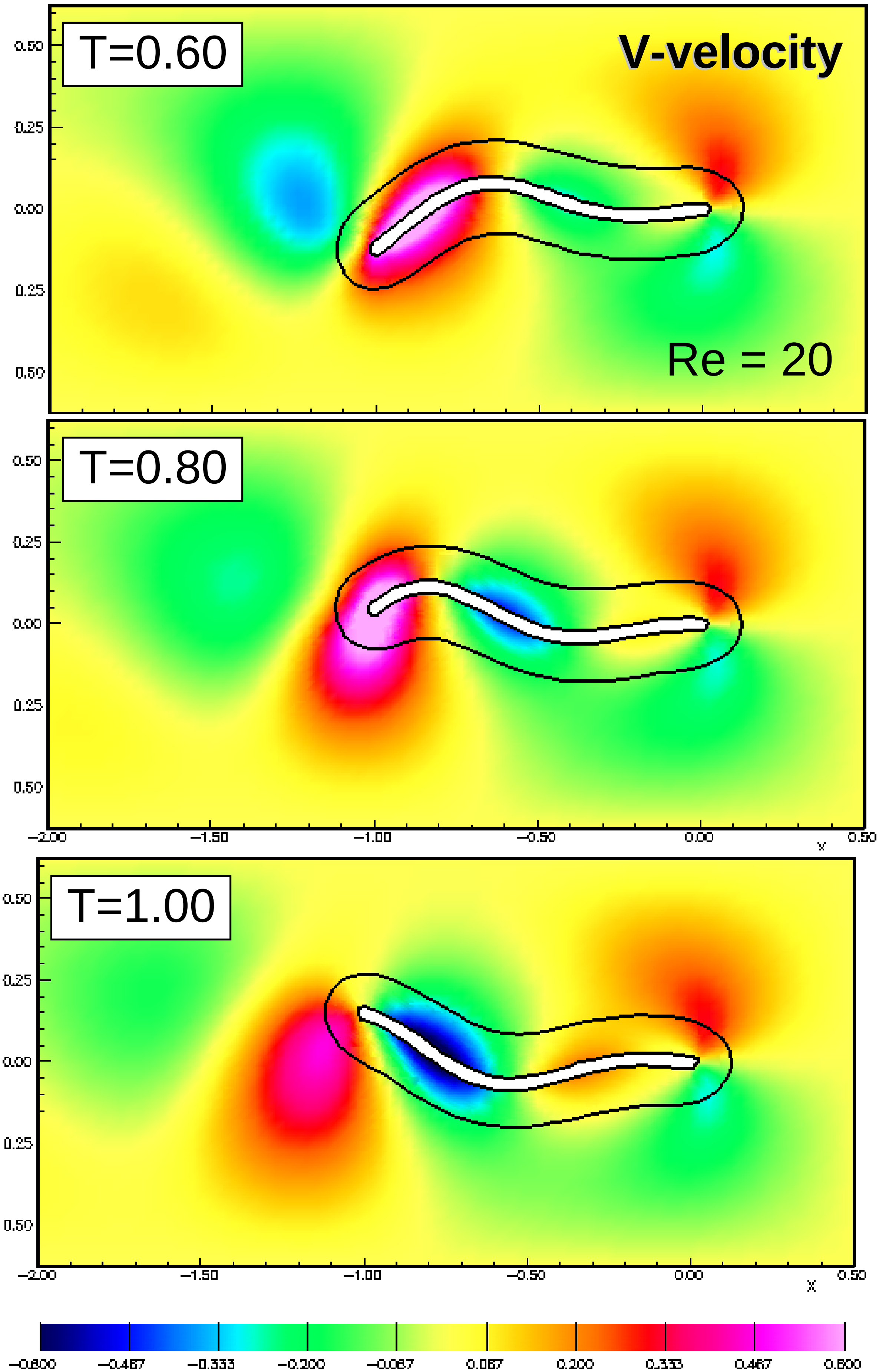
Kinematic BC $$\mathbf{U}(\mathbf{r}, t) = \dot{\mathbf{G}}(\mathbf{r}, t)$$

Pressure BC
$$\begin{aligned} \partial_n P &= -\underline{\mathbf{n} \cdot \ddot{\mathbf{G}}} + \nu \mathbf{n} \cdot \tilde{\Delta} \mathbf{U} \\ \nabla \cdot \mathbf{U} &= 0 \end{aligned}$$

- Implemented in ‘OverBlown’

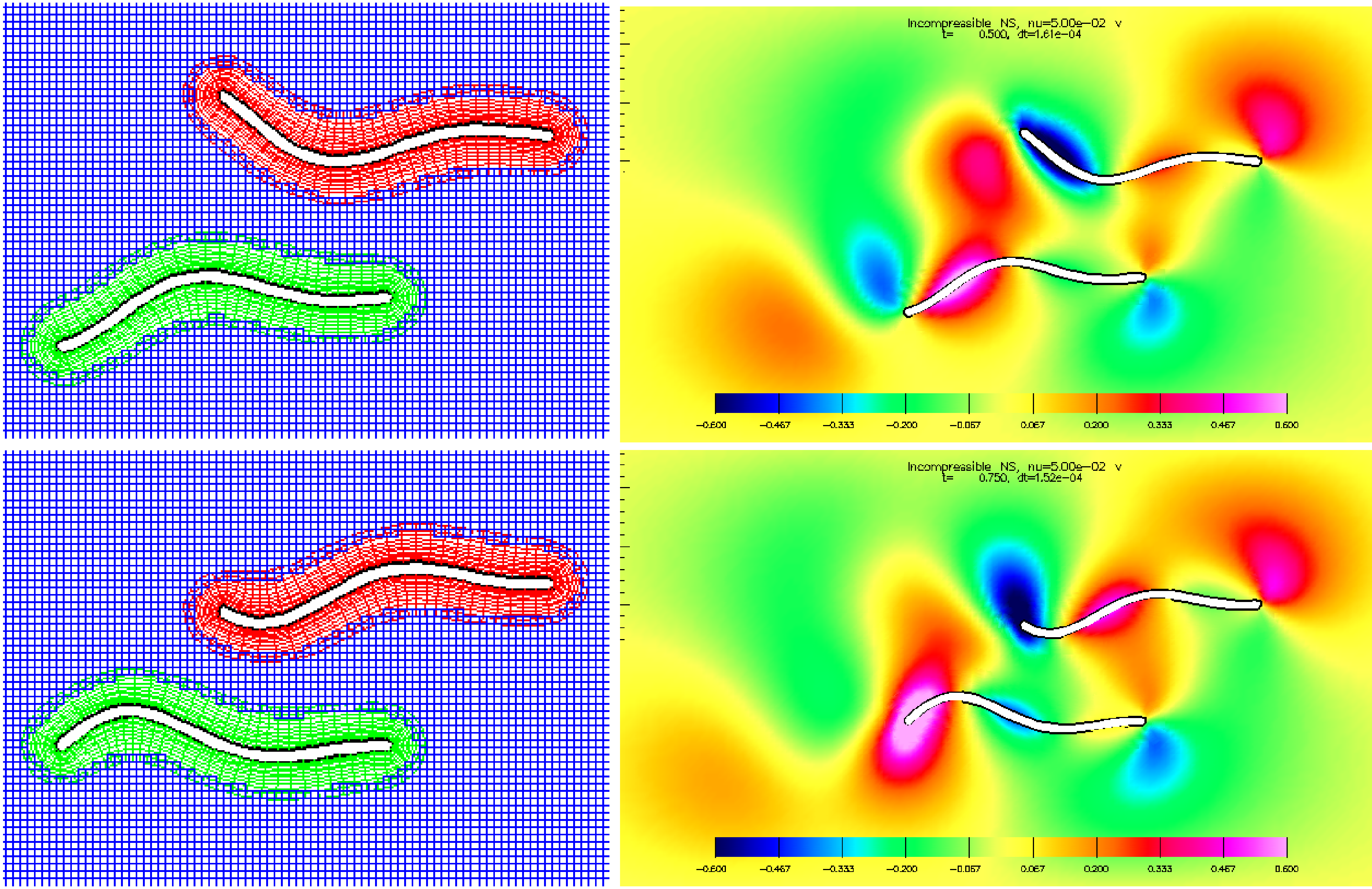
Overset grid discretization of $\Delta p = f$

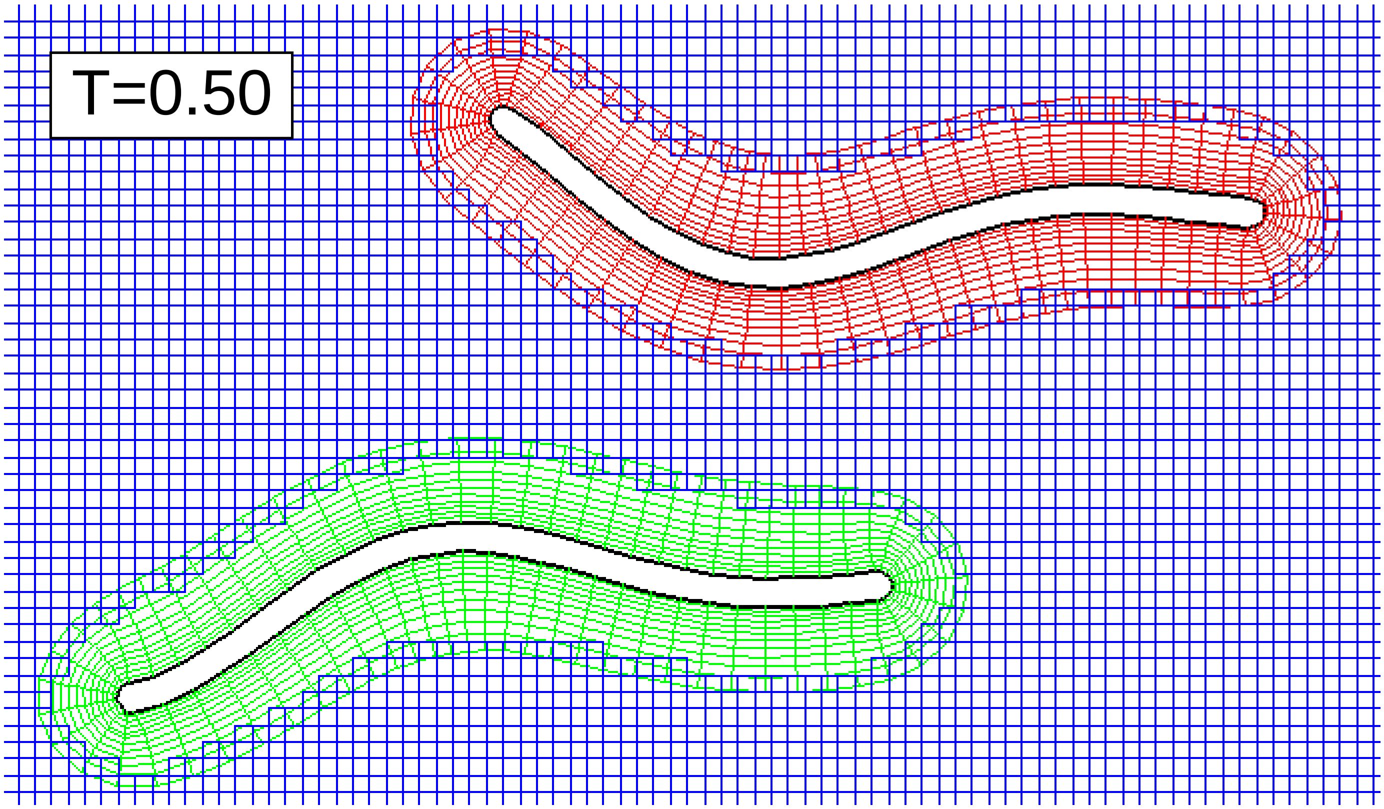
- ***Non-conservative formulation***
- ***2nd order finite differences, vertex centered***
- ***Biquadratic, implicit interpolation***
  - **Small overlap, 2nd order accuracy**
- ***Note:***
  - **System changes at each timestep**
  - **Resulting sparse system is *nonsymmetric***
  - ***Cannot* use direct methods**
  - ***Cannot* use CG**
- **New: overset grid multigrid (Henshaw & Chesshire '87)**
  - **Looking at using this for the pressure solve (WIP)**
  - **A novel formulation simplifies implementation (Henshaw)**
- **Reference: e.g. [Chesshire & Henshaw, '90]**

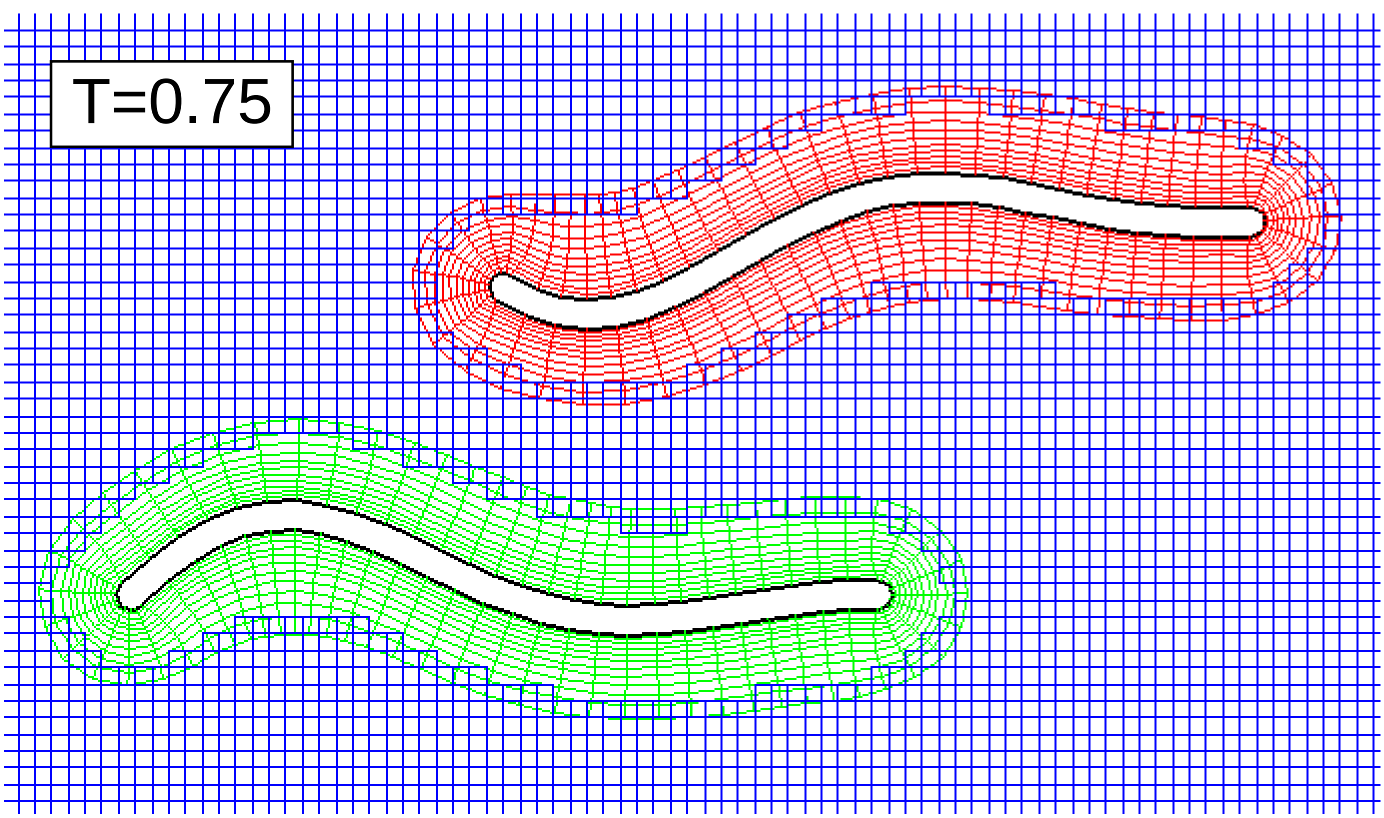


T=0.60
V-velocity
Re = 20
T=0.80
T=1.00
0.50
0.25
0.00
−2.00
−1.50
−1.00
−0.50
0.00
0.50
X
−0.600
−0.467
−0.333
−0.200
−0.067
0.067
0.200
0.333
0.467
0.600

Two filaments: prescribed dynamics




T=0.50


T=0.75

Conclusions

✓ Moving overset grid method for interface dynamics:

- ***Thin body-fitted grid conforms to the moving boundary***
- ***Most of the grid is Cartesian & fixed***
- ***Allows efficient structured grid finite difference schemes***

✓ Navier-Stokes solver on deforming, overset grids:

✓ Ongoing work

- Couple elastic dynamics for the boundary to the flow
- Compare with quasi-2D soapfilm experiments

New method should allow simulation of high Reynolds number biological flows

Acknowledgements

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- Mike Shelley

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